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A cyclodissipativity characterization of power factor compensation of nonlinear loads under nonsinusoidal conditions

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SUMMARY

Recently, it has been established that power factor (PF) improvement for nonlinear loads with nonsinusoidal source voltage is equivalent to imposing the property of cyclodissipativity to the source terminals. Using this framework, the classical capacitor and inductor compensators were interpreted in terms of energy equalization. The purpose of this brief note is to extend this approach in three directions. In the result reported in the literature, the supply rate is a function of the load, which is usually unknown, stymieing the applicability of the technique for compensator synthesis. Our first contribution is a new cyclodissipativity condition, which is also equivalent to PF improvement, but whose supply rate is now function of the compensator. Second, we consider general lossless linear compensators, instead of only capacitive or inductive compensators. As a result, we show that the PF is improved if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the load is ensured. Finally, we exhibit the gap between the ideal compensator, namely the one that achieves unitary PF, and the aforementioned equalization condition. This result naturally leads to the formulation of a problem of optimization of the parameters of the compensator. Copyright © 2011 John Wiley & Sons, Ltd.

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KEY WORDS: power factor; nonsinusoidal; cyclodissipativity; nonlinear; passive power filter

1. INTRODUCTION

One of the most significant current discussions in electrical engineering is the definition of the reactive power under nonsinusoidal conditions. New definitions of reactive power have been discussed in the last 30 years in the engineering literature. Although the mechanism of electric energy flow for nonsinusoidal and/or unbalance conditions is well understood today, there is not yet available a generalized power theory that can provide a simultaneous common base for energy billing, evaluation of power quality, detection of the major sources of waveform distortion, and theoretical calculations for the design of mitigation equipment such as active filters or dynamic compensators [1].

In power quality, optimizing energy transfer from an alternating current (ac) source to a load is a classical problem. The power factor (PF), defined as the ratio between the real or the active power (average of the instantaneous power) and the apparent power (the product of rms values of the voltage and current), captures the energy-transmission efficiency for a given load. The standard approach to improve the PF is to place a lossless compensator between the source and the load.

If the load is scalar linear time-invariant (LTI) and the generator is ideal—that is, with negligible impedance and fixed sinusoidal voltage—it is well known that the optimal compensator minimizes

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the phase shift between the source voltage and current waveforms [2, 3]. In addition, the generalization of the reactive power definition and the concept to a sinusoidal balanced three-phase system is clear [4]. However, the task of designing compensators that aim at improving PF for nonlinear time-varying loads operating in nonsinusoidal regimes is, on the other hand, far from clear.

The effectiveness of capacitive compensation in systems with nonsinusoidal voltages and currents has been widely studied, see e.g. [5, 6]. In [7], it has been shown that capacitive compensation may not be effective for nonsinusoidal voltages. Therefore, a more complex compensator is required in such situations. Furthermore, most of the approaches used to improve PF are based on *ad hoc* definitions of reactive power, [6], and a lack of consensus on these definitions produces misunderstanding of power phenomena in circuits with nonsinusoidal voltages and currents. Naturally, this generalization has been the subject of many discussions and proposals. See also [8, 9] for a more recent account of the field and [10] for a detailed description of several common misconceptions.

In the recent paper [9], a new framework for analysis and design of (possibly nonlinear) PF compensators for electrical systems operating in nonsinusoidal (but periodic) regimes with nonlinear time-varying loads was presented. This framework proceeds from the aforementioned, universally accepted, definition of PF and does not rely on any axiomatic definition of reactive power. It is shown that PF is improved if and only if the compensated system satisfies a certain cyclo-dissipativity property [11].

This result has been applied in [12] to analyze passive compensation of a classical half-bridge controlled rectifier with nonsinusoidal source voltage. Unfortunately, the supply rate in [9] depends explicitly on the load, which is typically unknown. Hence, the result cannot be used for compensator synthesis. One contribution of our work is the proof that PF improvement can also be characterized in terms of a new cyclo-dissipativity property where the supply rate is independent of the load and is solely determined by the compensator—paving the road for compensator design applications.

In [9], the case of LTI capacitive or inductive compensation is studied, showing that PF improvement is equivalent to energy equalization. In this work, we extend this result to consider arbitrary LTI lossless filters, and prove that for general lossless LTI filters, the PF is reduced if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the load is ensured. A final contribution of our work is to exhibit the gap between the ideal compensator, e.g. the one that achieves unitary PF, and the aforementioned equalization condition. This result naturally leads to the formulation of a problem of optimization of the compensator topology and its parameters.

2. A NEW CYCLODISSIPATIVITY CHARACTERIZATION OF PF COMPENSATION

2.1. Framework

We consider the energy transfer from an n -phase ac generator to a load, as Figure 1. It is an n -phase system with $n + 1$ wires where the first n are referenced either to a common ground, or the $(n + 1)$ th ('neutral') wire. The voltage and current of the source are denoted by the column vectors $v_s(t), i_s(t) \in \mathbb{R}^n$ and the load is described by a (possibly nonlinear and time varying) n -port network \mathfrak{N} (each port is formed by two terminals for which one is either the common ground or the neutral wire). We make the following assumptions.

Assumption 1

All signals are assumed to be periodic[‡] and have finite power; that is, they belong to

$$\mathcal{L}_2^n = \left\{ x : [0, T) \rightarrow \mathbb{R}^n : \|x\|^2 := \frac{1}{T} \int_0^T |x(\tau)|^2 d\tau < \infty \right\}, \quad (1)$$

[‡]Such waveforms can be represented by a summable Fourier series, viz. $x(t) = \sum_{\ell=-\infty}^{\infty} X_\ell \exp(j\ell\omega t)$ where the vector Fourier coefficients X_ℓ are given by $X_\ell = (1/T) \int_T x(t) \exp(-j\ell\omega t) dt$. Although, with the development of switching circuits and pulse technique, it becomes common to generate and process square waves, using Fourier series we can write an ideal square wave as an infinite series of many sine and cosine waves with different frequencies.

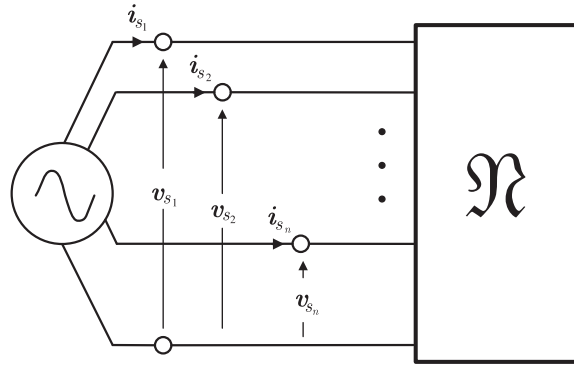


Figure 1. Illustrating power delivered to a (possibly nonlinear and time varying) load from an n -phase ac ideal generator.

where $|\cdot|$ is the Euclidean norm and the inner product in \mathcal{L}_2^n is defined as

$$\langle x, y \rangle := \frac{1}{T} \int_0^T x^\top(t) y(t) dt. \quad (2)$$

Assumption 2

The source is ideal, in the sense that v_s remains unchanged for all loads.

The universally accepted definition of PF is given as [1]:

Definition 1

The PF of the source is defined by

$$\text{PF} := \frac{P}{S}, \quad (3)$$

where

$$P := \langle v_s, i_s \rangle, \quad (4)$$

$$S := \|v_s\| \|i_s\| \quad (5)$$

are the active (real) power, also called average power [13], and the apparent power, respectively.

From Equation (3) and the Cauchy–Schwartz inequality, it follows that $P \leq S$. Hence, $\text{PF} \in [-1, 1]$ is a dimensionless measure of the energy-transmission efficiency. Cauchy–Schwartz inequality also tells us that a necessary and sufficient condition for the apparent power to equal the active power is that v_s and i_s are collinear—see Lemma 3.1 in [14]. If this is not the case, $P < S$ and compensation schemes are introduced to maximize the PF.

2.2. The PF compensation problem

The PF compensation configuration considered in the paper is depicted in Figure 2, where $Y_c, Y_\ell : \mathcal{L}_2^n \rightarrow \mathcal{L}_2^n$ are the admittance operators of the compensator and the load \mathfrak{N} , respectively. That is,

$$i_c = Y_c(v_s), \quad i_\ell = Y_\ell(v_s), \quad (6)$$

where $i_c, i_\ell \in \mathcal{L}_2^n$ are the compensator and load currents, respectively. In the simplest LTI case the operators Y_c, Y_ℓ can be described by their admittance transfer matrices, which we denote by $\hat{Y}_c(s), \hat{Y}_\ell(s) \in \mathbb{R}^{n \times n}(s)$, respectively.

The uncompensated PF, that is, the value of PF when $Y_c = 0$, is clearly given by

$$\text{PF}_u := \frac{\langle v_s, i_\ell \rangle}{\|v_s\| \|i_\ell\|}. \quad (7)$$

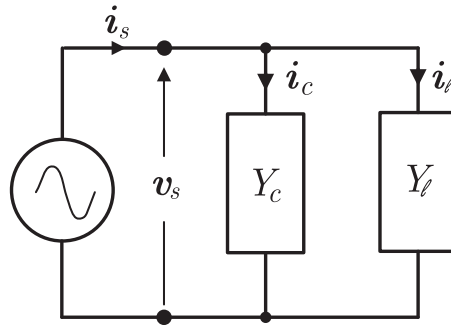


Figure 2. Schematic diagram of shunt PF compensation configuration.

Following standard practice, we consider only *lossless compensators*, that is,

$$\langle Y_c(v_s), v_s \rangle = 0 \quad \forall v_s \in \mathcal{L}_2^n. \quad (8)$$

We recall that, if Y_c is a passive LTI network, Equation (8) is equivalent to

$$\hat{Y}_c(j\omega) = -\hat{Y}_c^\top(-j\omega) \quad (9)$$

for all $\omega \in \mathbb{R}$ for which $j\omega$ is not a pole of \hat{Y}_c . Furthermore, if Y_c is a one-port passive LTI LC-network, losslessness implies

$$\text{Re}\{\hat{Y}_c(j\omega)\} = 0, \quad (10)$$

for all $\omega \in \mathbb{R}$ for which $j\omega$ is not a pole of \hat{Y}_c and all alternate zeros and poles are simple and lie on the imaginary axis and where $\text{Re}\{\hat{Y}_c(j\omega)\}$ is the real part of the admittance transfer $\hat{Y}_c(j\omega)$, see [15, 16].

2.3. PF compensation and cyclodissipativity

Dissipativity provides us with a useful tool for the analysis of nonlinear systems, which relates nicely to Lyapunov and \mathcal{L}_2 stability [17–19]. In accordance with physical concepts, a system is called dissipative if it does not produce energy, in some abstract sense. Typical examples of dissipative systems are: passive electrical networks, mechanical systems, viscoelastic materials, etc. Before proceeding with cyclodissipativity, we first present the definition of passivity, since this is the most commonly used.

Definition 2

The n -port system of Figure 1 is said to be passive if and only if

$$\int_0^T v_s^\top(t) i_s(t) dt \geq -\beta, \quad (11)$$

for all admissible $(v_s, i_s) \in \mathcal{L}_2^n \times \mathcal{L}_2^n$ and for all $T \geq 0$, where β is a positive constant that takes into account initial conditions.

Generally speaking, passivity means that the system does not produce energy itself and any increase of energy is externally supplied or the system is passive if we can extract from it only a finite amount of energy, which corresponds to the one that was initially stored. Examples of passive systems (in a theoretic and idealized setting) are motors, air conditioners, etc.

The concept of cyclodissipativity is inspired by the fact that cyclodissipative systems exhibit a dissipative behavior in cyclic motions. As explained in [20], cyclodissipativity is understood in terms of the available generalized energy. The idea is borrowed from thermodynamics, where the notion is formulated in a conceptually clearer manner than in circuits and systems theory.

Thermodynamical systems define cyclopassive systems as do, for example, less ‘physical’ systems as electrical networks with positive resistors and capacitors and inductors with either sign. The definition of cyclopassivity involves a function called supply rate, $w(v_s, i_s): \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$, which is locally integrable for every $v_s \in \mathbb{R}^n$ [11].

Definition 3

Given a mapping $w: \mathcal{L}_2^n \times \mathcal{L}_2^n \rightarrow \mathbb{R}$, the n -port system of Figure 1 is cyclopassive with respect to the supply rate $w(v_s, i_s)$ if and only if

$$\int_0^T w(v_s(t), i_s(t)) dt > 0, \quad (12)$$

for all $(v_s, i_s) \in \mathcal{L}_2^n \times \mathcal{L}_2^n$.

Notice that for cyclopassivity, the supply rate function $w(v_s, i_s)$ is of a specific form, namely, $w(v_s, i_s) := v_s^\top(t) i_s(t)$.

Remark 1

In words, a system is cyclopassive when it cannot create (abstract) energy over closed paths. It might, however, produce energy along some initial portion of such a trajectory. If so, it is cyclopassive, but not dissipative. Passivity is a special type of dissipativity and cyclopassivity. On the other hand, every dissipative (passive) system are cyclopassive. Loads as generators and batteries are not passive, and also not necessarily cyclopassive.

To place our result in context, and make the paper self-contained, we recall the following results from [9].

Proposition 1

Consider the system of Figure 2 with fixed Y_ℓ . The compensator Y_c improves the PF if and only if the system is cyclopassive with respect to the supply rate

$$w(v_s, i_s) := (Y_\ell(v_s) + i_s)^\top (Y_\ell(v_s) - i_s). \quad (13)$$

The proof follows from Equation (8) and the fact that the compensator is lossless.

The next result follows from Proposition 1 and it characterizes the set of all compensators Y_c that improve the PF for a given Y_ℓ .

Corollary 2

Consider the system of Figure 2. Then Y_c improves the PF for a given Y_ℓ if and only if Y_c satisfies

$$2\langle Y_\ell(v_s), Y_c(v_s) \rangle + \|Y_c(v_s)\|^2 < 0 \quad \forall v_s \in \mathcal{L}_2^n. \quad (14)$$

Dually, given Y_c , the PF is improved for all Y_ℓ that satisfy Equation (14).

Remark 2

The key advantage of cyclopassivity is that it restricts the set of inputs of interest to those that generate periodic solutions (a feature that is intrinsic in PF compensation problems) and it furthermore deals with ‘abstract’ energies.

The supply rate function (13) depends explicitly on the load. However, if the load is unknown, the result cannot be used for compensator synthesis. In the next subsection it is shown that the PF improvement can also be characterized by a new cyclopassivity property.

2.4. A new cyclopassivity condition for PF compensation

We underscore the fact that the supply rate (13) explicitly depends on the, usually unknown, load admittance Y_ℓ . Hence, the result of Proposition 1 can only be used for analysis of a given known load—as done in [12] for a TRIAC controlled rectifier. The proposition below gives an alternative

cyclodissipative characterization of PF improvement where the supply rate depends instead on the compensator admittance.

Proposition 3

Consider the system of Figure 2 with fixed Y_c . The PF is improved for all Y_ℓ such that the system is cyclodissipative with respect to the supply rate

$$w(v_s, i_s) := |Y_c(v_s)|^2 - 2i_s^\top Y_c(v_s). \quad (15)$$

Proof

We have shown above that $\text{PF} > \text{PF}_u$ if and only if $\|i_s\|^2 < \|i_\ell\|^2$. Using the fact that $i_s = i_c + i_\ell$, the latter inequality can be written as

$$\|i_c + i_\ell\|^2 < \|i_\ell\|^2, \quad (16)$$

which is equivalent to

$$\|i_c\|^2 + 2\langle i_c, i_\ell \rangle < 0. \quad (17)$$

Substituting $i_\ell = i_s - i_c$ in Equation (17) yields

$$\|i_c\|^2 - 2\langle i_c, i_s \rangle > 0. \quad (18)$$

The proof is completed replacing $i_c = Y_c(v_s)$. \square

As indicated above, the interest of the new cyclodissipativity property is that now the supply rate (15) depends on Y_c , that is to be designed. Current research is under way to exploit this new cyclodissipativity property to synthesize PF compensators.

3. WEIGHTED POWER EQUALIZATION AND PF COMPENSATION FOR RLC LOADS

In this section we extend Proposition 5 in [9], where the PF compensators are assumed to be capacitors or inductors, to general lossless LTI filters. Similar to [9], we assume that the load is a nonlinear RLC circuit consisting of lumped dynamic elements (n_L inductors, n_C capacitors) and static elements (n_R resistors). Capacitors and inductors are defined by the physical laws and constitutive relations [13]:

$$i_C = \dot{q}_C, \quad v_C = \nabla H_C(q_C), \quad (19)$$

$$v_L = \dot{\phi}_L, \quad i_L = \nabla H_L(\phi_L), \quad (20)$$

respectively, where $i_C, v_C, q_C \in \mathbb{R}^{n_C}$ are the capacitors currents, voltages, and charges, and $i_L, v_L, \phi_L \in \mathbb{R}^{n_L}$ are the inductors currents, voltages, and flux-linkages, $H_L: \mathbb{R}^{n_L} \rightarrow \mathbb{R}$ is the magnetic energy stored in the inductors, $H_C: \mathbb{R}^{n_C} \rightarrow \mathbb{R}$ is the electric energy stored in the capacitors, and ∇ is the gradient operator. We assume that the energy functions are twice differentiable and for linear capacitors and inductors,

$$H_C(q_C) = \frac{1}{2} q_C^\top C^{-1} q_C, \quad (21)$$

$$H_L(\phi_L) = \frac{1}{2} \phi_L^\top L^{-1} \phi_L, \quad (22)$$

respectively, with $L \in \mathbb{R}^{n_L \times n_L}$, $C \in \mathbb{R}^{n_C \times n_C}$. To avoid cluttering the notation, we assume that L, C are diagonal matrices.

Finally, we distinguish between two sets of nonlinear static resistors: n_{R_i} current-controlled resistors and n_{R_v} voltage-controlled resistors, for which the characteristics are given by the following one-to-one real-valued functions:

$$v_{R_i} = \hat{v}_{R_i}(i_{R_i}), \quad (23)$$

and

$$i_{R_v} = \hat{i}_{R_v}(v_{R_v}), \quad (24)$$

respectively, where $i_{R_i}, v_{R_i} \in \mathbb{R}^{n_{R_i}}$ are the currents, voltages of the current-controlled resistors, and $i_{R_v}, v_{R_v} \in \mathbb{R}^{n_{R_v}}$ are the currents, voltages of the voltage-controlled resistors, with $n_R = n_{R_i} + n_{R_v}$.

Recalling the definition of real power (4), we introduce the following.

Definition 4

Given a compensator admittance Y_c , the weighted (real) power of a single-phase circuit with port variables $(v, i) \in \mathcal{L}_2 \times \mathcal{L}_2$ is given by

$$P^w := \langle Y_c(v), i \rangle. \quad (25)$$

If Y_c is LTI

$$P^w = \sum_{k=-\infty}^{\infty} \hat{Y}_c[k] \hat{V}[k] \hat{I}^*[k], \quad (26)$$

where $\hat{V}[k], \hat{I}[k]$ are the k th spectral lines of v and i , respectively, and $\hat{Y}_c[k] := \hat{Y}_c(k\omega_0)$, with $\omega_0 := 2\pi/T$. That is, P^w is the sum of the power components of the circuit modulated by the frequency response of Y_c —hence the use of the ‘weighted’ qualifier.[§]

The aforementioned definition motivates the next lemma.

Lemma 4

Consider a nonlinear time-invariant (TI) current-controlled {voltage-controlled} one-port resistor characterized by Equation (23) {(24)}, and a fixed LTI lossless compensator Y_c with $n = 1$. Let $\hat{Y}_c(j\omega)$ denote the associated admittance transfer function. If $\hat{Y}_c(j\omega)$ has a zero at the origin, then the weighted averaged power along periodic trajectories satisfies

$$P_{R_i}^w := \langle Y_c(v_{R_i}), i_{R_i} \rangle = 0, \quad (27)$$

$\{P_{R_v}^w := \langle Y_c(v_{R_v}), i_{R_v} \rangle = 0\}$ for all admissible pair $(v_{R_i}, i_{R_i}) \in \mathcal{L}_2 \times \mathcal{L}_2$ $\{(v_{R_v}, i_{R_v}) \in \mathcal{L}_2 \times \mathcal{L}_2\}$, and for all $\omega \in \mathbb{R}$ for which $j\omega$ is not a pole of $\hat{Y}_c(j\omega)$.

Proof

From the Foster’s reactance theorem, see [21, 22], the impedance function of LTI lossless can be written in the form

$$\hat{Z}(s) = \frac{g(s^2 + \omega_{z_1}^2)(s^2 + \omega_{z_2}^2) \dots}{s(s^2 + \omega_{p_1}^2)(s^2 + \omega_{p_2}^2) \dots}, \quad (28)$$

where $g > 0$ and $0 \leq \omega_{z_1} < \omega_{p_1} < \omega_{z_2} < \omega_{p_2} \dots$. Furthermore, ω_{z_1} can be zero or not depends upon whether $\hat{Z}(s)$ has a zero or a pole at the origin. We have that $\hat{Y}_c(s) = 1/\hat{Z}(s)$. Since Y_c admits a factorization $Y_c = Y_{c_1}(Y_{c_2})$, then

$$\langle i_{R_i}, Y_c(v_{R_i}) \rangle = \langle i_{R_i}, Y_{c_1}(Y_{c_2}(v_{R_i})) \rangle, \quad (29)$$

$$\langle i_{R_v}, Y_c(v_{R_v}) \rangle = \langle i_{R_v}, Y_{c_1}(Y_{c_2}(v_{R_v})) \rangle = \langle i_{R_v}, Y_{c_2}(Y_{c_1}(v_{R_v})) \rangle, \quad (30)$$

where we used the fact that Y_{c_1} and Y_{c_2} commute.[¶] For a lossless n -ports, we have that Y_c is skew Hermitian, i.e. $\hat{Y}_c(s) + \hat{Y}_c^*(s) = 0$ for all $s = j\omega$, where Y_c^* is the adjoint (or the conjugate

[§]Since the spectral lines of real signals satisfy $\hat{F}[-k] = \hat{F}^*[k]$, the weighted power is a real number.

[¶]Since Y_{c_1} and Y_{c_2} are two continuous, linear, TI operators, there is an invertible operator S such that $SY_{c_1}S^{-1} = H_1$ and $SY_{c_2}S^{-1} = H_2$, where H_1 and H_2 denote multiplication operators. Since H_1 and H_2 commute, Y_{c_1} and Y_{c_2} commute.

transpose) of Y_c , see [22, 23]. Consider the case of the nonlinear TI current-controlled resistor. By the assumption that $\hat{Y}_c(s)$ has a zero at the origin, we define $\hat{Y}_{c1}(s) = s$ and thus we have

$$\langle i_{R_i}, Y_c(v_{R_i}) \rangle = \langle Y_{c1}^*(i_{R_i}), Y_{c2}(v_{R_i}) \rangle. \quad (31)$$

Since Y_{c1} is skew-Hermitian, and $Y_{c1} = d/dt$, the last expression become

$$\langle i_{R_i}, Y_c(v_{R_i}) \rangle = - \left\langle \frac{di_{R_i}}{dt}, Y_{c2}(\hat{v}_{R_i}(i_{R_i})) \right\rangle, \quad (32)$$

and the right-hand side of Equation (32) can be written as

$$\left\langle \frac{di_{R_i}}{dt}, Y_{c2}(\hat{v}_{R_i}(i_{R_i})) \right\rangle = \frac{1}{T} \int_0^T (Y_{c2}(\hat{v}_{R_i}(i_{R_i})) \frac{di_{R_i}}{dt}) dt. \quad (33)$$

By substitution, we obtain

$$\left\langle \frac{di_{R_i}}{dt}, Y_{c2}(\hat{v}_{R_i}(i_{R_i})) \right\rangle = \frac{1}{T} \int_{i_{R_i}(0)}^{i_{R_i}(T)} Y_{c2}(\hat{v}_{R_i}(i_{R_i})) di_{R_i}. \quad (34)$$

Since the input is periodic with period T , i.e. $i_{R_i}(0) = i_{R_i}(T)$, then the inner product (32) is zero. The convolution $Y_{c2}\hat{v}_{R_i}(i_{R_i})$ is also periodic with period T in steady state, see Theorem 4.1.2 in [24]; and the existence and uniqueness of the composition can be proved by the Volterra series, see Theorem 3.2.1 in [24]. An analogous result holds for the case of the nonlinear TI voltage-controlled resistor, i.e. $\langle Y_{c2}(\hat{i}_{R_v}(v_{R_v})), dv_{R_v}/dt \rangle = 0$. \square

Remark 3

By using Parseval's theorem, it is easily verified that in the case of an LTI one-port resistor^{||}, the necessary condition that the associated admittance transfer function $\hat{Y}_c(j\omega)$ had a zero at the origin can be removed.

Now, let us use Lemma 4 and Remark 3 to prove the following main result.

Proposition 5

Consider the system of Figure 2 with $n=1$,** a full nonlinear RLC load and a fixed LTI lossless compensator Y_c with admittance transfer function $\hat{Y}_c(j\omega)$ which has a zero at the origin.

(i) PF is improved if and only if

$$\frac{1}{2} V_s^w + \sum_{q=1}^{n_L} P_{L_q}^w + \sum_{q=1}^{n_C} P_{C_q}^w < 0, \quad (35)$$

where V_s^w is the rms value of the filtered voltage source, that is,

$$V_s^w := \|Y_c v_s\|^2 = \sum_{k=1}^{\infty} |\hat{Y}_c(k) \hat{V}_s(k)|^2, \quad (36)$$

$$P_{C_q}^w := \sum_{k=-\infty}^{\infty} \hat{Y}_c[k] \hat{V}_{C_q}[k] \hat{I}_{C_q}^*[k], \quad (37)$$

$$P_{L_q}^w := \sum_{k=-\infty}^{\infty} \hat{Y}_c[k] \hat{V}_{L_q}[k] \hat{I}_{L_q}^*[k] \quad (38)$$

are the weighted powers of the q th capacitor and inductor, respectively.

^{||}A linear resistor is both current- and voltage-controlled and is represented by $u_{R_i} = R i_{R_i}$ (Ohm's law), where R is the resistance, or, similarly, $i_{R_v} = G v_{R_v}$, where $G (= R^{-1})$ is the conductance.

**This condition is imposed, without loss of generality, to simplify the presentation of the result.

(ii) Condition (35) may be equivalently expressed as

$$\left\langle \left(\frac{1}{p} Y_c \right) v_L, \nabla^2 H_L v_L \right\rangle - \left\langle i_C, \left(\frac{1}{p} Y_c \right) \nabla^2 H_C i_C \right\rangle > \frac{1}{2} V_s^w, \quad (39)$$

where $p := d/dt$.

(iii) If the capacitors and inductors are linear, their weighted powers become

$$\begin{aligned} P_{C_q}^w &:= 2\omega_0 \sum_{k=1}^{\infty} \left\{ k \operatorname{Im}\{\hat{Y}_c[k]\} \sum_{q=1}^{n_C} C_q |\hat{V}_{C_q}[k]|^2 \right\}, \\ P_{L_q}^w &:= -2\omega_0 \sum_{k=1}^{\infty} \left\{ k \operatorname{Im}\{\hat{Y}_c[k]\} \sum_{q=1}^{n_L} L_q |\hat{I}_{L_q}[k]|^2 \right\}, \end{aligned} \quad (40)$$

where $\operatorname{Im}\{\hat{Y}_c[k]\}$ is the imaginary part of the admittance transfer function $\hat{Y}_c[k]$.

(iv) Furthermore, the results (i)–(iii) can be extended for a general LTI lossless compensator, if the resistors of the load are LTIs.

Proof

Corollary 2 shows that the PF is improved if and only if Equation (14) holds, which may be equivalently expressed as

$$\|Y_c(v_s)\|^2 + 2\langle Y_c(v_s), i_\ell \rangle < 0. \quad (41)$$

Applying the generalized form of Tellegen's theorem to the RLC load, one gets

$$i_\ell^\top Y_c v_s = i_{R_v}^\top Y_c v_{R_v} + i_{R_i}^\top Y_c v_{R_i} + i_L^\top Y_c v_L + i_C^\top Y_c v_C, \quad (42)$$

see [25], which upon integration yields

$$\langle i_\ell, Y_c(v_s) \rangle = \langle i_L, Y_c(v_L) \rangle + \langle i_C, Y_c(v_C) \rangle, \quad (43)$$

where we have used the fact that, because of Lemma (4), $\langle i_{R_v}, Y_c(v_{R_v}) \rangle = 0$ and $\langle i_{R_i}, Y_c(v_{R_i}) \rangle = 0$ for nonlinear LTI resistors. The proof of the Condition (35) is completed by substituting the expression above in Equation (41) and computing $\|Y_c(v_s)\|^2$ directly from Definition 4.

Now, the second claim is established as follows:

$$\langle i_L, Y_c v_L \rangle = \langle \nabla H_L, Y_c \dot{\phi}_L \rangle \quad (44)$$

$$= - \left\langle \nabla^2 H_L v_L, \left(\frac{1}{p} Y_c \right) v_L \right\rangle, \quad (45)$$

where the first identity follows from the relations (20) and the second uses the well-known property of periodic functions $\langle f, \dot{g} \rangle = -\langle \dot{f}, g \rangle$. Similar derivations with the term $\langle i_C, Y_c v_C \rangle$ yield Equation (39).

To prove (iii) we use Equation (26), the basic relations for LTI inductors and capacitors

$$\hat{I}_{C_q}[k] = jk\omega_0 C_q \hat{V}_{C_q}[k], \quad (46)$$

$$\hat{V}_{L_q}[k] = jk\omega_0 L_q \hat{I}_{L_q}[k], \quad (47)$$

and the fact that Y_c satisfies Equation (9).

Finally, the proof of iv) follows directly from Remark (3). \square

Remark 4

Condition (35) indicates that the PF will be improved if and only if the overall weighted power (supplied plus stored) is negative.

Remark 5

From Equation (39) (or replacing Equation (40) in Equation (35)) we see that PF improvement is equivalent to average power equalization between inductors and capacitor—notice the minus signs—with the gap being determined by the weighted supplied power.

4. IDEAL PF COMPENSATION

We now use the framework presented in the previous section to explore the power transmission efficiency. In particular, we give two conditions to achieve unitary PF: the first one is only necessary, whereas the second one is necessary and sufficient. Although both conditions can be derived from standard considerations, giving them in the framework used in the paper allows, on one hand, to identify the gap between PF improvement, characterized in Proposition 1, and achieve unitary PF. On the other hand, with these conditions we can formulate a compensator synthesis problem—as illustrated in the next section.

Proposition 6

Consider the system of Figure (2) with fixed Y_ℓ and a lossless compensator Y_c . A necessary condition to achieve unitary PF is

$$\|i_c\|^2 + \langle i_c, i_\ell \rangle = 0. \quad (48)$$

Proof

From the definition of PF, Equation (3), and the Cauchy–Schwarz inequality, it follows that unity PF is achieved if and only if $i_s(t)$ and $v_s(t)$ are co-linear, i.e. $i_s(t) = \alpha v_s(t)$, for some nonzero constant α . Since the compensator is lossless, we have

$$0 = \langle i_c, v_s \rangle \quad (49)$$

$$= \alpha \langle i_c, v_s \rangle = \langle i_c, \alpha v_s \rangle. \quad (50)$$

Hence, $\langle i_c, i_s \rangle = 0$, which means that i_c is orthogonal to i_s . Now, replacing $i_s = i_\ell + i_c$ in the condition above, we obtain the desired result. \square

We have shown in Proposition 1 that the PF is improved if and only if Equation (17), which we repeat here for the ease of reference,

$$\|i_c\|^2 + 2\langle i_c, i_\ell \rangle < 0 \quad (51)$$

holds. Comparing Equation (48) with Equation (51), we notice that there is a gap between PF improvement and optimality, which stems from the fact that Equation (48) clearly implies Equation (51). Referring to Figure 3, we have a (rather obvious) geometric interpretation of this gap.^{††} While the PF improvement condition (51) ensures that $\|i_s\| < \|i_\ell\|$, the optimality condition (48) places i_s orthogonal to i_c .

The proposition below, which follows directly from the proof of Proposition 1, gives a necessary and sufficient condition for optimal power transfer.

Proposition 7

Consider the system of Figure 2 with fixed Y_ℓ . The lossless compensator Y_c yields a unitary PF if and only if

$$\langle v_s, i_\ell \rangle = \|v_s\| \|i_s\|. \quad (52)$$

^{††}It is important to recall that orthogonality in Figure 3 should be understood with respect to the inner product in \mathcal{L}_2^n , not the Euclidean space—the geometric interpretations should take this into consideration.

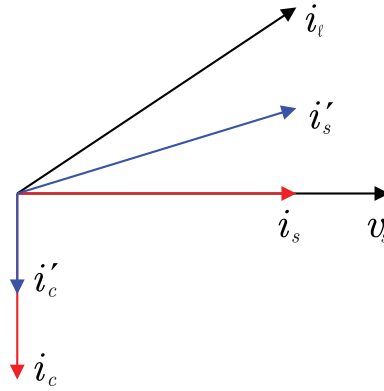


Figure 3. Geometric interpretation of power-factor compensation. The currents i'_c, i'_s satisfy the condition of improved power-factor, Equation (51), and the currents i_c, i_s satisfy the necessary condition for unity power-factor, Equation (48).

Proof

From Kirchhoff's current law $i_s = i_c + i_\ell$, the relation $i_c = Y_c(v_s)$, and from the lossless condition (8), it follows that $\langle v_s, i_s \rangle = \langle v_s, i_\ell \rangle$. Consequently, Equation (3) becomes

$$\text{PF} = \frac{\langle v_s, i_\ell \rangle}{\|v_s\| \|i_s\|}. \quad (53)$$

Condition (52) follows directly setting $\text{PF} = 1$. \square

The result has, again, a very simple interpretation, which also clarifies why condition (48), although necessary, is not sufficient for optimal power transfer. Indeed, it is clear that although there are many source currents i_s co-linear with v_s , there is only one that ensures Equation (52). This current is actually well known in the power systems community and it is known as Fryze's current, defined by

$$i_F(t) = \frac{\langle i_\ell, v_s \rangle}{\|v_s\|^2} v_s(t). \quad (54)$$

5. APPLICATION OF THE PROPOSED FRAMEWORK

In this section, we present two examples that illustrate the points discussed in the paper.

Example 1 (Series LTI RLC load)

In 1932, Fryze [26] introduced a procedure to design LTI PF compensators for LTI loads with nonsinusoidal (periodic) sources. The underlying idea is to find a PF compensator that achieves unitary PF for each of the harmonics of the source. More precisely, assume $v_s(t)$ contains $N \in \mathbb{Z}_+$ spectral lines at frequencies ω_i , $i = 1, \dots, N$. Then, design (if possible) the PF compensator such that

$$\text{Im}\{\hat{Y}_c(j\omega_i) + \hat{Y}_\ell(j\omega_i)\} = 0. \quad (55)$$

In this way, for each harmonic component of $v_s(t)$, the compensated load behaves as a pure resistor.

In [6], the following illustrative example was introduced to reveal the limitations of Fryze's approach. Consider the LTI series RLC circuits of Figure 4 supplied with a periodic nonsinusoidal voltage source

$$v_s(t) = 100\sqrt{2}(\sin t + \sin 3t) \text{ V}. \quad (56)$$

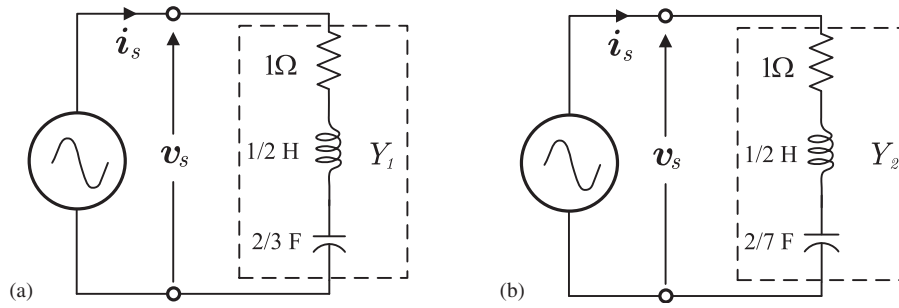


Figure 4. Two loads that cannot be distinguished in terms of Fryze's active and reactive powers.

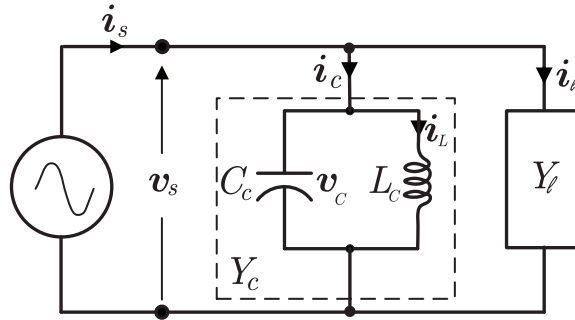


Figure 5. Parallel LC compensation and load.

Consider two loads with $R_\ell = 1 \Omega$, $L_\ell = \frac{1}{2} \text{ H}$, and two different values for the capacitor $C_{\ell_1} = \frac{2}{3} \text{ F}$ and $C_{\ell_2} = \frac{2}{7} \text{ F}$. It can be shown that for both loads, the uncompensated PF is the same, and is given by $\text{PF}_u = 0.71$. Evaluating the load admittance for the first and the third harmonics, we get

$$\hat{Y}_{\ell_1}[1] = 0.5 + j0.5 \text{ S}, \quad \hat{Y}_{\ell_1}[3] = 0.5 - j0.5 \text{ S}, \quad (57)$$

for the first load, and

$$\hat{Y}_{\ell_2}[1] = 0.1 + j0.3 \text{ S}, \quad \hat{Y}_{\ell_2}[3] = 0.9 - j0.3 \text{ S}, \quad (58)$$

for the second one.

Now, consider as compensator the parallel LC filter of Figure 5. Some simple calculations show that for both loads, there exists values of C_c and L_c such that Equation (55) holds, namely $L_c = \frac{4}{3} \text{ H}$ and $C_c = \frac{1}{4} \text{ F}$ for the first load, and $L_c = \frac{20}{9} \text{ H}$ and $C_c = \frac{3}{20} \text{ F}$ for the second one. However, as some calculations show, while ideal PF compensation is achieved for the first load, i.e. $\text{PF} = 1$, for the second load $\text{PF} = 0.78$.

Let us now explain the difference between the two loads using the conditions derived in the previous section. Toward this end, define the function

$$f_n(C_c, L_c) := \|i_c\|^2 + \langle i_c, i_\ell \rangle, \quad (59)$$

that, for the given voltage source, is of the form

$$\begin{aligned} f_n(C_c, L_c) = & \sum_{k=1,3} \frac{1 - (k\omega_0)^2 L_c C_c}{L_c} \{L |\hat{I}_L(k)|^2 - C |\hat{V}_C(k)|^2\} \\ & + \sum_{k=1,3} \left| \frac{1 - (k\omega_0)^2 L_c C_c}{k\omega_0 L_c} \right|^2 |\hat{V}_s(k)|^2, \end{aligned} \quad (60)$$

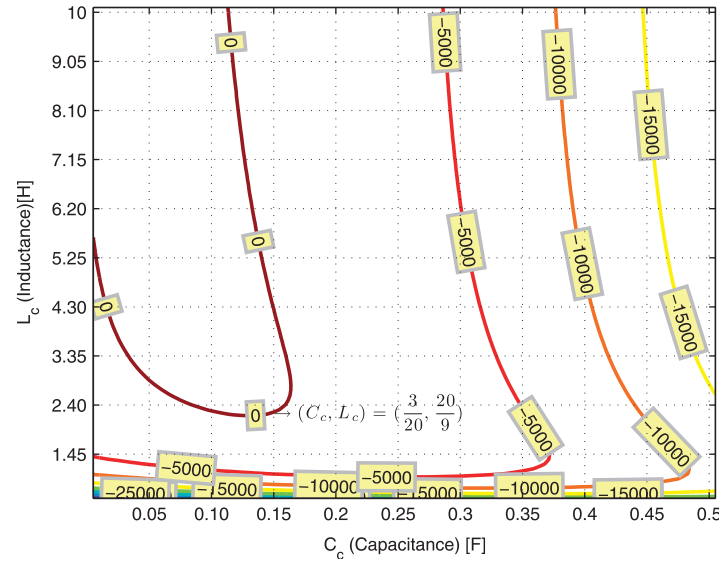


Figure 6. The plot of level sets of $-f_n(C_c, L_c)=0$.

where $\hat{V}_s(k)$ is the k th spectral line of $v_s(t)$ and $\hat{V}_{C_\ell}(k)$ and $\hat{I}_{L_\ell}(k)$ are the spectral lines of the capacitor voltage and inductor current load. Recall that in Proposition 6 it was shown that a necessary condition to achieve unitary PF is that $\|i_c\|^2 + \langle i_c, i_\ell \rangle = 0$. Figure 6 shows the plot of the level sets of $-f_n(C_c, L_c)$ for the second load, which shows that there indeed exists a set of compensator parameters (C_c, L_c) such that $f(C_c, L_c)=0$ —this set clearly includes the values $L_c = \frac{20}{9}$ H and $C_c = \frac{3}{20}$ F. However, since this condition is only necessary, there is no guarantee that PF=1. To verify the latter, we define a second function

$$f_{ns}(C_c, L_c) := \|i_s\| \|v_s\| - \langle i_\ell, v_s \rangle, \quad (61)$$

that, in view of Proposition 7, is equal to zero if and only if PF=1. The graph of this function, shown in Figure 7 proves that there is no set of parameters (C_c, L_c) such that $f_{ns}(C_c, L_c)=0$. Hence, it is not possible to achieve unitary PF for the second load with the compensator of Figure 5.

It is actually possible to prove that there does not exist an LTI lossless compensator—of any topology—that will yield PF=1 for the second load. Indeed, for an arbitrary LTI lossless compensator, the admittance of the compensated load, at a frequency $k\omega_0$, is of the form

$$\hat{Y}_\ell(jk\omega_0) + \hat{Y}_c(jk\omega_0) = \frac{R_\ell}{R_\ell^2 + \left(\frac{1 - (k\omega_0)^2 L_\ell C_\ell}{k\omega_0 C_\ell} \right)^2} + j \left\{ \frac{\frac{1 - (k\omega_0)^2 L_\ell C_\ell}{k\omega_0 C_\ell}}{R_\ell^2 + \left(\frac{1 - (k\omega_0)^2 L_\ell C_\ell}{k\omega_0 C_\ell} \right)^2} - Y_c(k\omega_0) \right\}, \quad (62)$$

where we recall that a lossless compensator affects only the imaginary part of the overall admittance. With $\omega_0=1$, the first load \hat{Y}_{ℓ_1} has the same conductance—i.e. $\text{Re}\{\hat{Y}_\ell(jk\omega_0)\}$ —for the fundamental and the third harmonic, see Equation (57). However, this conductance changes for the second load \hat{Y}_{ℓ_2} , see Equation (58). Consequently, there is no shunt lossless compensator that will make the source current be proportional to the voltage as needed for unitary PF.

Example 2 (Single-phase half semi-controlled bridge rectifier)

Consider the classical single-phase semi-converter-controlled rectifier load, terminated by a resistor in Figure 8. In [12], it is shown that the load can be modeled as a linear time-varying resistor

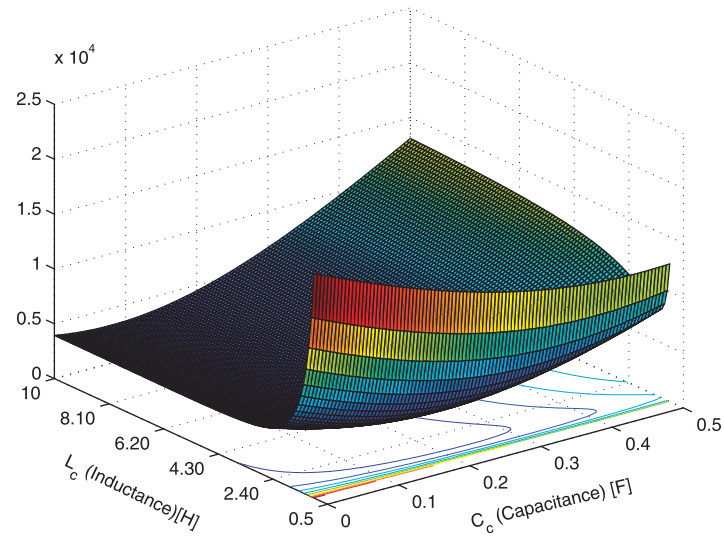
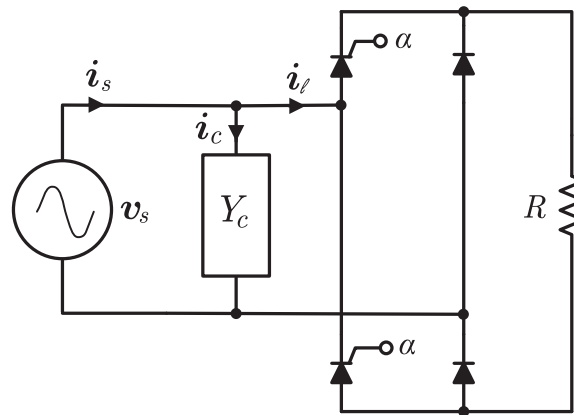
Figure 7. Plot of the function $f_{ns}(C_c, L_c)$.

Figure 8. Single-phase half semi-controlled bridge rectifier with a linear resistor load.

under the following assumptions:

- (A1) $v_s(t)$ is periodic.
- (A2) $v_s(t)$ changes sign every half period, i.e. $v_s(kT) = v_s(kT/2) = 0$.
- (A3) $v_s(t)$ is nonnegative in the first half-period and nonpositive in the second one, that is,

$$v_s(t) \begin{cases} \geq 0, & t \in \left[0, \frac{T}{2}\right], \\ \leq 0, & t \in \left[\frac{T}{2}, T\right]. \end{cases} \quad (63)$$

- (A4) The SCR's firing angle, α , is constant and $\alpha < T/2$.

In this case, the admittance operator of the load is given by

$$i_\ell(t) = \begin{cases} 0 & \text{if } t \in \left[\frac{kT}{2}, \frac{kT}{2} + \alpha\right), \quad k=0, 1, \dots, \\ \frac{v_s(t)}{R} & \text{otherwise.} \end{cases} \quad (64)$$

In [12], it has been proved that a capacitive compensator, $Y_c = C_c p$, improves the PF, for all v_s satisfying Assumptions A1–A3, if and only if $C_c < C_c^{\max}$, where

$$C_c^{\max} = \frac{T}{4\pi^2 R} \frac{v_s^2(\alpha) + v_s^2\left(\frac{T}{2} + \alpha\right)}{\sum_{n=-\infty}^{\infty} n^2 |\hat{V}(n)|^2}. \quad (65)$$

We attract the readers attention to the qualifier ‘for all’. Interestingly, a similar result is also given for inductive compensation, which is a rather surprising fact.

The previous result only gives a bound on the admissible value of the capacitance to improve the PF, but does not tell us how to select the optimal value. To achieve this objective, we now use the framework of our paper and formulate an optimization problem based on the necessary and sufficient condition for PF improvement of Proposition 3, namely, $\|i_c\|^2 + 2\langle i_c, i_\ell \rangle < 0$. We define the function

$$f_o(C_c) := \|i_c\|^2 + 2\langle i_c, i_\ell \rangle, \quad (66)$$

which for $v_s = V_s \sin \omega_0 t$ becomes

$$f_o(C_c) = C_c^2 \omega_0^2 V_s^2 - \frac{C_c}{RT} \left[v_s^2(\alpha) + v_s^2\left(\frac{T}{2} + \alpha\right) \right]. \quad (67)$$

This function is quadratic in the unknown C_c and takes its minimal value at

$$C_c^* = \frac{T}{8\pi^2 R |\hat{V}_s(1)|^2} \left[v_s^2(\alpha) + v_s^2\left(\frac{T}{2} + \alpha\right) \right]. \quad (68)$$

To check the validity of our calculations, we compute the PF as a function of the compensators capacitance, which is given by

$$\text{PF}(C_c) = \frac{\frac{1}{2} - \frac{1}{\pi}}{\sqrt{(2RC_c\omega_0)^2 - \frac{4}{\pi}RC_c\omega_0 + 2\left(1 - \frac{1}{\pi}\right)}}. \quad (69)$$

Figure 9 shows the plot of $\text{PF}(C_c)$ for the parameters $R = 10 \, \Omega$, $\alpha = 3\pi/4 \, \text{s}$ and $v_s(t) = 280 \cos(100\pi t) \, \text{V}$. For these values, the uncompensated PF is $\text{PF}_u = 0.3014$. As shown in the plot,

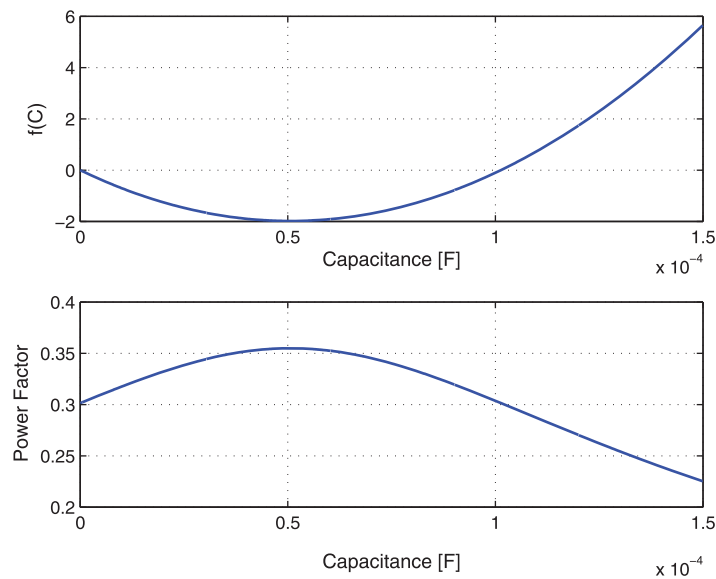


Figure 9. The plots of the function $f_o(C_c)$ and the power-factor $\text{PF}(C_c)$.

the maximum is achieved at $C_c^* = 50.8 \mu\text{F}$ and takes the value $\text{PF}(C_c^*) = 0.3549$, confirming the derivations above.

6. CONCLUSION

In this paper, the framework for analysis of PF compensation for nonsinusoidal nonlinear networks based on cyclodissipativity introduced in [9] has been extended in several directions. First, a new cyclodissipativity characterization of PF improvement was introduced. Second, we have proved that the PF is improved with a general lossless LTI filter if and only if a certain equalization condition between the weighted powers of compensator and load is ensured. Third, the gap between the ideal compensator, i.e. the one that achieves unitary PF, and the one that satisfies the aforementioned equalization condition was described. Finally, through two examples we have illustrated that the results reported here can be used for the formulation of a problem of optimization of the compensator.

Some issues that remain open, and are currently being explored, include:

- We have used the conditions of Propositions 6 and 7 to formulate an optimization problem for the compensator parameters. Also, we have revealed the existence of a gap between the cyclodissipativity condition for PF improvement and the necessary condition to achieve $\text{PF} = 1$ of Proposition 6. However, it is not clear whether we can use the cyclodissipativity condition to design the PF compensator. More precisely, it is not clear whether minimizing the function $\|i_c\|^2 + 2\langle i_c, i_\ell \rangle$ will yield an optimal PF. In the second example of Section 5, this turns out to be true.
- A key assumption to develop the framework is that the source is ideal, that is, that its impedance can be neglected. In many applications, this is not the case. Some preliminary derivations prove that this assumption can be relaxed, preserving the essential features of the framework. This result will be reported elsewhere in the near future.
- Although we here concentrate on passive shunt compensation, we aim at extending the results to active filters. In this respect, we are studying the use of power electronic converters in nonsinusoidal situations [27, 28].

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